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A NOVEL METHOD FOR MAGNETOHYDRODYNAMIC SIMULATIONS AND ITS FIRST APPLICATIONS IN ASTROPHYSICS AND COSMOLOGY ON HIGH PERFORMANCE COMPUTATIONAL SYSTEMS

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Summary Magnetic fields are one of the most important phenomena in science and engineering, as they are present on almost every scale in nature, ranging from atomic magnetic moments to the intergalactic space, and are used in applications ranging from Magnetic Resonance Imaging to nuclear fusion.

In this work we first present a novel powerful method for high performance magnetohydrodynamic (MHD) calculations which is based on kinetic schemes. In particular, using it, it is possible to derive the MHD equations directly from the Boltzmann Equation without the necessity of an *ad hoc* introduction of terms related to electromagnetic interactions.

With that at hand, we were then able to apply the method to one of the most important problems in present day astrophysics and cosmology, namely to the question of the origin and time evolution of Intergalactic Magnetic Fields. As for their origin, there are mainly two scenarios discussed in the literature – on the one hand the cosmological one, where the magnetic field is produced by some process in the very early Universe, and on the

other hand the cosmological one, where a seed of the magnetic field is created during structure formation and then amplified by some dynamo effect.

Here, we show first results of the aforementioned application of our method – on the one hand, concerning the astrophysical scenario, the simulation of galactic winds, i.e. the ejection of matter from galaxies which might also carry magnetic energy, and on the other hand, for the cosmological scenario, the time evolution of primordial magnetic fields and their possible imprints on the Cosmic Microwave Background (CMB).

1 INTRODUCTION

The interaction of magnetic fields and matter via the Lorentz Force is one of the most fundamental phenomena in physics. If conducting fluids and gases are considered, the corresponding branch of physics is called Magnetohydrodynamics (MHD) and is basically bringing together the equations of fluid dynamics (continuity and momentum equations together with energy conservation) and electromagnetism (Maxwell's Equations).

The fluid dynamics quantities and equations can be derived from the distribution function f which is governed by the Boltzmann equation which, without external forces, reads

$$\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t} + \boldsymbol{\xi}\cdot\nabla f(\mathbf{x},\boldsymbol{\xi},t) = \mathcal{C}f(\mathbf{x},\boldsymbol{\xi},t)\,, \tag{1}$$

where $\mathbf{x}$ and $\boldsymbol{\xi}$ are the position and velocity vector, respectively, and $\mathcal{C}f$ is the collision integral.

The relevant quantities can be obtained by taking the so-called moments of $f(\mathbf{x},\boldsymbol{\xi},t)$, i.e. by calculating the integral

$$\mathcal{M}_i f(\mathbf{x},\boldsymbol{\xi},t) = \iiint \phi_i(\boldsymbol{\xi}) f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi\,, \tag{2}$$

using the invariants $\phi_i = (1,\boldsymbol{\xi},\xi^2/2)$, therefore obtaining

$$\mathcal{M}_1 f(\mathbf{x},\boldsymbol{\xi},t) = \iiint \phi_1(\boldsymbol{\xi}) f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \iiint 1 f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \frac{\rho}{m}\,, \tag{3}$$

$$\mathcal{M}_2 f(\mathbf{x},\boldsymbol{\xi},t) = \iiint \phi_2(\boldsymbol{\xi}) f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \iiint \boldsymbol{\xi} f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \frac{\rho}{m}\mathbf{u}\,, \tag{4}$$

$$\mathcal{M}_3 f(\mathbf{x},\boldsymbol{\xi},t) = \iiint \phi_3(\boldsymbol{\xi}) f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \iiint \frac{\xi^2}{2} f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \frac{\mathfrak{e}}{m}\,, \tag{5}$$

where ρ is the mass density, m is the particle mass, $\mathbf{u}$ is the velocity field and $\mathfrak{e}$ is the (total) energy density.

Solving the general form of the Boltzmann Equation, (1), is a rather complicated task which is usually done numerically. However, for the collisionless case, i.e. for $\mathcal{C}f(\mathbf{x},\boldsymbol{\xi},t)$, an analytical solution can be found, given by

$$f_{\mathrm{M}}(\mathbf{x}, \boldsymbol{\xi}, t) = \frac{\rho m^{\frac{1}{2}}}{[2\pi k_{\mathrm{B}} T]^{\frac{3}{2}}} \exp\left\{ -\frac{m}{2k_{\mathrm{B}} T(\mathbf{x}, t)} [\boldsymbol{\xi} - \mathbf{u}]^2 \right\}, \tag{6}$$

where k_{B} is the Boltzmann Constant and T is the temperature.

On the other hand, the electromagnetic contribution is given by Ohm's Law,

$$\mathbf{j} = \sigma [\mathbf{E} + \mathbf{u} \times \mathbf{B}] , \tag{7}$$

where σ is the (local) conductivity and $\mathbf{j}$ is the electric current density, Faraday's Law,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{8}$$

Ampére's Law,

$$\nabla \times \mathbf{B} = \mu \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \tag{9}$$

and the monopole condition of the magnetic field,

$$\nabla \cdot \mathbf{B} = 0 . \tag{10}$$

As it turns out, combining the fluid and electromagnetic equations in a natural and consistent way is rather difficult. Usually, it is done by an *ad hoc* introduction of electromagnetic terms, resulting in, for example, the Vlasov Equation [1].

In this work we present a novel method for the derivation of the MHD equations in a more consistent way by introducing complex velocities, such that all electromagnetic terms can be derived directly from the Boltzmann equations. We then proceed to develop an algorithm to solve the resulting equations numerically and apply it to problems in astrophysics and cosmology.

In particular we are interested in the physics of Intergalactic Magnetic Fields (IGMF), i.e. the magnetic fields in the voids between galaxies and galaxy clusters [2]. Since it is difficult to measure them, little is known about their origin, evolution and even magnitude, such that a lot of effort has to be put into investigating them.

These difficulties arise due to the scales which are involved. On the one hand, being present on the largest scales of the Universe and being created in each early stages (s. below), while potentially being created on much smaller scales, numerical simulations of the phenomenon are difficult as they have to involve such large range of spacial and temporal scales. There are mainly two ways to solve this problem: Either carrying out (semi)analytical computations [3, 4, 5, 6] which, due to their nature, drastically reduce the computational time, or performing large scale numerical MHD simulations [7, 8, 9, 10] which may not have the full range, but, with the increasing computational power, are getting more and more detailed and feasible for extrapolation.

On the other hand, being thought to have a very small magnitude of nG or below [2], IGMF are difficult to measure experimentally since in any cosmic object (Galaxies, stars, etc.) they are overlaid by this object's own much stronger magnetic, hence being undetectable. Therefore, one has to measure them inside the voids or to rely on more indirect methods. The former is, for example, done by cosmic (and gamma) ray observation, since they (or their parent particle, respectively) are charged and hence are subject to the Lorentz Force resulting in imprints of the IGMF seen in observations [11, 12, 13, 14]. The latter is done for example by calculating the impact of the IGMF on some cosmological parameters and then comparing this predicted value to measurements, hence being able to set constraints (see [15] and the references therein). However, it should be noted that while several well-established *upper* limits exist, the *lower* limits, derived from the first method, are still under debate [2], hence making the zero IGMF hypothesis a conclusion which has not been ruled out yet.

Finally, another important, but still not fully known aspect of IGMF is their origin. Currently there are two scenarios being discussed in the literature: On the one hand the Cosmological Scenario, where the IGMF are created in the very early Universe in a cosmological process such as Inflation [16, 17, 18, 19] or a cosmological phase transition (for example the electroweak [20, 21, 22, 23, 24] or the quantum chromodynamic [25, 26, 27, 28] one). Since it is a *cosmological* process, strong small-scale magnetic fields are created all over the universe and then decay, thus transporting energy to larger scales, resulting in the present day large-scale IGMF. On the other hand there is the Astrophysical Scenario, where the magnetogenesis is thought to be happening later in time, during structure formation, for example due to protogalactic density perturbation [29], cosmic ray currents [30] or galactic outflows [31].

This work is structured as follows: After explaining the novel method of complex distribution functions which allow it to derive the MHD equation in an elegant way in Sec. 2, we show, in Sec. 3, two of its applications for EGMF, namely the simulation of galactic winds (Sec. 3.1), and some first results for cosmological simulations of EGMF with their implications for CMB observations (Sec. 3.2).

2 A NOVEL METHOD FOR MAGNETOHYDRODYNAMIC SIMULATIONS

2.1 Complex Distribution Function

In this work we propose a new model to derive the MHD equations by introducing a complex-valued distributian function [32, 33, 34] which in the equilibrium case is given by

$$f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t) = \frac{\rho m^{\frac{1}{2}}}{\left[2\pi k_{\mathrm{B}}T\right]^{\frac{3}{2}}} \exp\left\{-\frac{m}{2k_{\mathrm{B}}T(\mathbf{x},t)}\left[\boldsymbol{\xi}-\Big(\mathbf{u}+i\mathbf{v}_{\mathrm{A}}\Big)\right]^{2}\right\}\,, \tag{11}$$

where $\mathbf{v}_{\mathrm{A}}$ is the Alfven Velocity given by

$$\mathbf{v}_{\mathrm{A}} = \frac{\mathbf{B}}{\sqrt{\mu\rho}}\,, \tag{12}$$

μ being the (local) magnetic permeability. As one can see, (11) maintains the general from of the real-valued equilibrium distribution function (6), while at the same time introducing the electromagnetic contributions.

This complex distribution function is an elegant, consistent and natural extension of the classical distribution function. Introducing the magnetic field into the distribution function in the way it is done above is motivated by several aspects. First of all, since magnetic fields cause a circular movement of charged particles due to the Lorentz Force, using complex variables is a natural and well-known way to take that fact into account, by that also considering the axial vector behaviour of the magnetic field [35]. Second, it has been proven that the concept of both the distribution function and the Boltzmann Equation remain valid even after the introduction of electromagnetic fields [36]. Finally, as can be shown by explicit calculations, the formalism of the section above remains valid if one generalizes Eq. (4) to

$$\Re\mathcal{M}_2 f(\mathbf{x},\boldsymbol{\xi},t) = \Re\iiint \phi_2(\boldsymbol{\xi}) f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \Re\iiint \boldsymbol{\xi} f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \frac{\rho}{m}\mathbf{u}\,, \tag{13}$$

$$\Im\mathcal{M}_2 f(\mathbf{x},\boldsymbol{\xi},t) = \Im\iiint \phi_2(\boldsymbol{\xi}) f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \Im\iiint \boldsymbol{\xi} f(\mathbf{x},\boldsymbol{\xi},t)\mathrm{d}^3\xi = \frac{\rho}{m}\mathbf{v}_{\mathrm{A}}\,, \tag{14}$$

where the operators $\Re$ and $\Im$ denote the real and imaginary part, respectively.

2.2 Balance Equation

After obtaining the distribution function including the electromagnetic contributions we now proceed to obtain the time evolution equations for the relevant MHD quantities.

This is done, in analogy with the procedure for the pure hydrodynamic case from [37], by considering the change of the distribution function (11) over a short period of time τ, which is taken to be the relaxation time, such that after this period of time the distribution function is in equilibrium again. We, hence, are able to carry out a Taylor expansion:

$$f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t+\tau) = \sum_{n=0}^{\infty}\frac{\tau^n}{n!}\frac{\partial^n f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)}{\partial t^n} \overset{(1)}{=} \sum_{n=0}^{\infty}(-1)^n\frac{\tau^n}{n!}\left(\boldsymbol{\xi}\cdot\nabla\right)^n f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)\,. \tag{15}$$

Considering only the first three terms on both sides we obtain

$$\begin{aligned} &f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t) + \tau\frac{\partial f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2 f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)}{\partial t^2} \\ &= f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t) - \tau\left(\boldsymbol{\xi}\cdot\nabla\right) f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t) + \frac{\tau^2}{2}\left(\boldsymbol{\xi}\cdot\nabla\right)^2 f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)\,, \end{aligned} \tag{16}$$

from which the hyperbolic differential time evolution,

$$\frac{\partial f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)}{\partial t} + \frac{\tau}{2}\frac{\partial^2 f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)}{\partial t^2} + \left(\boldsymbol{\xi}\cdot\nabla\right) f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t) = \frac{\tau}{2}\left(\boldsymbol{\xi}\cdot\nabla\right)^2 f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)\,, \tag{17}$$

may be derived, while considering only the first *two* terms on each side results in the parabolic equation

$$\frac{\partial f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+(\boldsymbol{\xi}\cdot\nabla)\,f_{\mathrm{M}}(\mathbf{x},\boldsymbol{\xi},t)=0\,. \tag{18}$$

Taking the moments $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{M}_3$, this results in the following system of equations [38, 39, 40]:

$$\frac{\partial\rho}{\partial t}+\frac{\tau}{2}\frac{\partial^2\rho}{\partial t^2}+\mathrm{div}\rho\,(\mathbf{u}-\mathbf{w})=0\,, \tag{19}$$

$$\frac{\partial\rho\mathbf{u}}{\partial t}+\frac{\tau}{2}\frac{\partial^2\rho\mathbf{u}}{\partial t^2}+\mathrm{div}\left[\rho\,(\mathbf{u}-\mathbf{w})\times\mathbf{u}+\frac{B_iB_k}{\mu}\right]+\nabla\left(p+\frac{B^2}{2\mu}\right)=\mathrm{div}P_{\mathrm{NS}}\,, \tag{20}$$

$$\frac{\partial E}{\partial t}+\frac{\tau}{2}\frac{\partial^2 E}{\partial t^2}+\mathrm{div}\left[\left(E+p+\frac{B^2}{2\mu}\right)(\mathbf{u}-\mathbf{w})\right]=\mathrm{div}\mathbf{q}+\mathrm{div}\left(P_{\mathrm{NS}}\mathbf{u}\right)\,, \tag{21}$$

$$\frac{\partial\mathbf{B}}{\partial t}+\frac{\tau_{\mathrm{m}}}{2}\frac{\partial^2\mathbf{B}}{\partial t^2}=\mathrm{rot}\left[(\mathbf{u}-\mathbf{w})\times\mathbf{B}+\nu_{\mathrm{m}}\mathrm{rot}\mathbf{B}\right] \tag{22}$$

$$\mathrm{div}\mathbf{B}=0\,, \tag{23}$$

where

$$w_k=\frac{1}{\rho}\frac{\partial}{\partial x_i}\left[\left(p+\frac{B^2}{2\mu}\right)\delta_{ik}+\rho u_iu_k-\frac{B_iB_k}{\mu}\right]\,, \tag{24}$$

P_{NS} is the viscous stress tensor, $\mathbf{q}$ is the heat flux vector, ν_{m} is the magnetic viscosity and τ_{m} is defined via the relation

$$\frac{\tau_{\mathrm{m}}}{2\rho}\left(p+\frac{B^2}{2\mu}\right)=\nu_{\mathrm{m}}\,. \tag{25}$$

3 APPLICATIONS

In this section we present first results of the application of the method described above to the problem of EGMF.

3.1 Simulating Galactic Winds

The simulations presented here are based on the idea that supernova (SN) explosions inside the galaxy produce enough energy to eject matter and energy from the galaxy. This phenomenon is what is usually known as Galactic Wind. Here we use the same initial conditions as in [41], namely a gas mass density profile given by

$$\rho(z)=\frac{\Sigma_{\mathrm{g}}}{2z_0\cosh^2\left(\frac{z}{z_0}\right)}\,, \tag{26}$$

where the z axis is pointing in the direction perpendicular to the galactic disk, Σ_{g} is the surface density of the gas in z direction, and z_0 is a measure for the distribution of mass along the z axis.

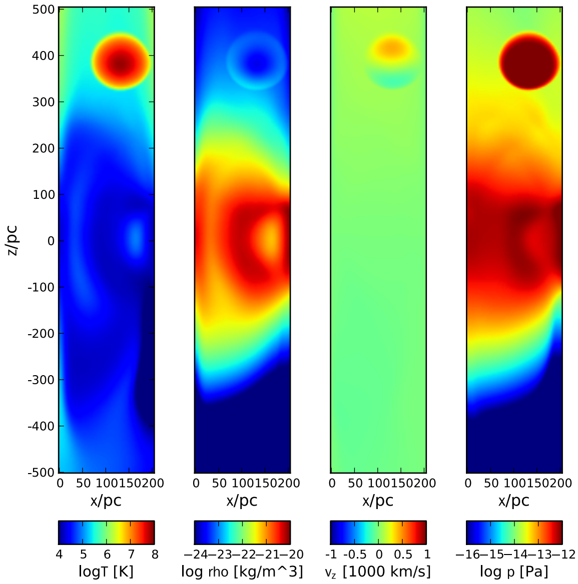

Figure 1: Simulation of Supernova-driven Galactic Wind after $t = 5\,\mathrm{Myr}$. From left to right: Temperature, gas mass density, velocity along the z axis and pressure.

The gravitational potential (which we assume to be static for now) is given by

$$\Phi(z) = \frac{2\pi G z_0 \Sigma_g}{\alpha_\mathrm{g}} \ln\cosh\left(\frac{z}{z_0}\right) , \tag{27}$$

where $\alpha_g = \rho/\rho_\mathrm{tot}$, i.e. the ratio of the gas mass density to the total mass density and G is the Gravitational Constant. With the assumption of initial hydrostatic equilibrium and the proton mass m_p, one gets $z_0 = \alpha_\mathrm{g} k_\mathrm{B} T_0 / (\pi m_\mathrm{p} G)$, where for the results presented here we have used $\alpha_\mathrm{g} = 0.1$, $\Sigma_\mathrm{g} = 11.6 M_\odot/\mathrm{pc}^2$ and $T_0 = 10^4\,K$.

The SN formation rate and energy were taken from [42], where they were found to be $\nu_1 = 1/330\mathrm{yr}^{-1}$ and $\nu_2 = 1/44\mathrm{yr}^{-1}$ for SN Type I and II, respectively, such that for the simulated time of $t_\mathrm{sim} = 5\,\mathrm{Myr}$ and the simulation box used ($256 \times 256 \times 1280$ nodes for the size $200\,\mathrm{pc} \times 200\,\mathrm{pc} \times 1000\,\mathrm{pc}$) the expected amount of SN is $N_\mathrm{NS} = 15.9$ [43, 44].

After some basic astrophysical simulations using the proposed new method have been carried out before [45], here we present the results of the first full-scale simulation of an actual physics problem for which the input parameters used were taken from observations. In Fig. 1 one can see a cross-section of the galaxy after $t = 5\,\mathrm{Myr}$. The results we obtain are in good agreement with previous results and will be used in the future to calculate how much of magnetic field energy is ejected along with the matter outflows.

3.2 Simulating the Time Evolution of Primordial Magnetic Fields and its impact on the Cosmic Microwave Background

Since in this case we want to simulate the Early Universe on the largest scales, the initial conditions are rather simple since we have to consider a isotropic and homogeneous

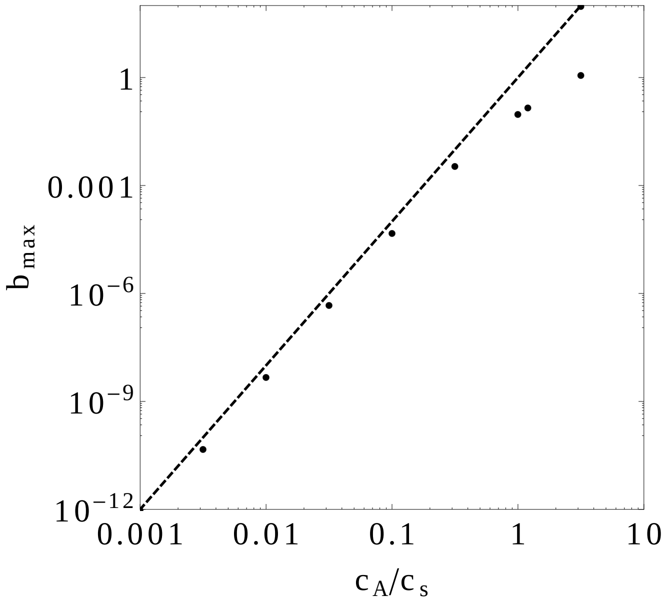

Figure 2: Dependence of the maximal clumping factor b on the ratio between the Alfven velocity c_A and the sound of speed c_s.

MHD system in an Expanding Universe. Furthermore, in order to take into account the interaction of the medium with free photons, we introduce a drag term $\propto \alpha\mathbf{v}$, where α is antiproportional to the photon mean free path [46]. Finally, the last component is a stochastic magnetic field $\mathbf{B}$.

Since a magnetic fields effectively acts as a pressure term, it causes density perturbations of the medium, which we measure by the so-called clumping factor b defined as

$$b = \left\langle \frac{\delta\rho}{\rho} \right\rangle^2 = \frac{1}{V} \frac{\int \rho(\mathbf{x}) - \langle\rho\rangle \, \mathrm{d}V}{\langle\rho\rangle} \tag{28}$$

As it turns out, from the size of the density perturbations, which are imprinted on the Cosmic Microwave background (CMB), one can deduce upper limits on the magnitude of the magnetic fields [47]. With the results from our simulations, presented in Fig. 2, we therefore were able to derive upper limits on the EGMF in the order of picogauss, which is better than most of the previous constraints by several orders of magnitude.

4 CONCLUSIONS AND OUTLOOK

In this paper we have presented a novel method for the derivation of the MHD equations directly from the distribution function by introducing complex velocities. We then applied it to two important problems in the physics of EGMF, namely galactic outflows and the imprints of the time evolution of EGMF on the CMB.

In the future we will explicitly include magnetic fields in the galactic wind simulations and compare them to our present estimates of the outflow of magnetic energy from the galaxy. This, in turn, can then be used to investigate the astrophysical scenario of EGMF magnetogenesis.

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